

an interest in the historical as well as the mathematical, there may be some discussion of how the thinkers of the time related mathematical concepts to the rest of the physical and social world. The joint mathematics meetings have had large and lively sessions on the history of mathematics for several years. The history of statistics is not accorded a similar level of respect in our curricula or at our meetings.

There are many obvious reasons for this difference. The subject is newer, there are fewer programs that lead to degrees in statistics, and statistics faculty are often housed in schools of engineering, agriculture, or business that have traditionally highlighted applications. Nonetheless, an interdisciplinary course in the history of statistics could be a great intellectual adventure for faculty and students alike. With college administrations and funding agencies increasingly emphasizing interdisciplinary work and quantitative literacy, such courses are likely to develop. Indeed, some may have already developed, though when I did an internet search on three key words (“statistics,” “history,” and “course”), I found only one (at the University of Chicago) among the first 30 hits. (A site related to the Master’s Golf tournament comes up first.)

Certainly one barrier to teaching courses in the history of statistics is that many of us do not know much about it. We may have read a little and we might know some stories about Bayes, Galton, Gossett, and Fisher that we relate cheerfully to classes when the work of these pioneers is discussed. The deeper intellectual history of our field is not a topic we routinely discuss. Contrast this with the mathematical community, where the philosophies of mathematicians from Euclid to the present are commonly invoked in arguments about the nature of proof and the value of the field. Our lesser emphasis on history seems a shame, because one thing that differentiates statistics from mathematics is the opportunity for scholarly argument. Unlike leading statistics journals, mathematics journals rarely have lively comments and rejoinders following articles. This engaging dialogue in the statistics literature often has a philosophical basis, but the language available to express these deeper ideas is limited. Reading *The Science of Conjecture* could serve as a valuable resource for statisticians who would like to engage in, and expand the scope of, our professional arguments.

For the majority of teachers unlikely to develop a full course in the history of statistics, Franklin has provided a reference that casts many great historical and philosophical events in language familiar to statisticians. I know that it will help me when I want to introduce the notions of type I and type II errors by discussing Pascal’s wager. Students and teachers in an actuarial science course could gain some perspective on their work by reading Chapter 10, on the development of insurance and annuities. This is a serious and scholarly work that I expect often will inform my teaching.

Despite of my overall positive feelings, a couple of caveats about the book are in order. First, the book is not easy. It is well written, sometimes humorous, and apparently meant to provoke good-natured argument; but it is chock full of references to people, places, and events that are unfamiliar to me. I suspect most of my technically trained colleagues will face the same difficulty, although we should not let that stop us (we expect our students to make that sort of effort all the time). A second warning is that the book’s breadth means that specialists in other fields would find much to argue with in the particulars of the applications discussed. When a book on the social sciences includes a short section on statistics, I almost always find some of the quantitative ideas presented are tangential, misleading, or wrong. Even with the impressive research presented, I would be surprised if Franklin’s treatment of some topics did not invoke similar reactions from specialists in other fields.

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### Mathematical Statistics With Mathematica.

Colin ROSE and Murray SMITH. New York: Springer-Verlag, 2002. ISBN 0-387-95234-9. xiii + 481 pp. (+2 CDs). \$79.97.

There is no doubt that the computer revolutionized applied statistics by relieving researchers (as well as teachers and students) of the burdens of tedious hand calculations. No similar revolution has occurred in mathematical statistics, perhaps because computer programs capable of handling such sophisticated mathematics do not lend themselves to statistical purposes. One capable program is Mathematica (Wolfram 1999). Mathematica is well known to have a flat learning curve (where time is the abscissa and accumulated

knowledge the ordinate). It is not so much a computer program as it is a way of life. Quite understandably, statisticians have shied away from learning and using Mathematica, and certainly have not given serious thought to inflicting it on their students.

With this book, Rose and Smith have remedied this deplorable situation. The book is not so much a book about Mathematica as it is a book about mathematical statistics brought to life by computer; it just so happens that their vehicle for doing so is Mathematica, an excellent choice. The centerpiece of the book is the Mathematica add-on mathStatica that Rose and Smith have written, which actually makes Mathematica easy to use.

Because the book is all about traditional mathematical statistics, it should come as no surprise that its table of contents subsumes much of a first-year graduate text (e.g., Hogg and Craig 1995), although some topics are omitted and there are frequent forays into more advanced areas. The prerequisites for this book, so the authors claim, are 1 year of mathematics or statistics and no knowledge of Mathematica—but they expect the person unacquainted with Mathematica to spend a couple of hours with a tutorial (e.g., it is important to know the difference between  $=$  and  $:=$  and to distinguish between  $x[1]$  and  $x[[1]]$ ). It is refreshing to find that the stated prerequisites are accurate.

Rose and Smith have modelled mathStatica on the way people really do statistics. The basis of the add-on is a handful of operators, most notably `Expect[x, f]`, `Prob[x, f]`, `Transform[eqn, f]`, and `PlotDensity[f]`. It might seem that a mere handful is insufficient, but consider the classic volumes by Stuart and Ord (1987, 1991); they do much of their work with just these four operators. With mathStatica, for example, once the discrete density  $f$  and its domain over  $x$  are defined, its probability generating function is computed simply by entering `pgf = Expect[t^x, f]`. It is remarkable how much can be done with just these four operators, and mathStatica has nearly a hundred operators.

The introduction (30 pp.) demonstrates mathStatica via some simple applications that require no programming knowledge, and also provides reliable installation instructions (installing on Linux is just as easy as installing on Windows). Note that mathStatica works only with Mathematica version 4 or higher; it will not work on version 3.

Chapter 2, “Continuous Random Variables” (50 pp.), covers moments and generating functions, including the characteristic function; conditioning, truncation, and censoring; and pseudo-random number generation for the continuous case. Example 24 derives the Black–Scholes equation via expectation rather than solving a partial differential equation, and it does so in only four lines.

Chapter 3, on discrete random variables (36 pp.), covers common discrete distributions, mixing distributions (distinguishing between parameter mixing and component mixing and providing examples of each), and pseudo-random number generation for the discrete case. With mathStatica, it is a simple matter to derive the negative binomial (Poisson-gamma), Pólya–Aeppli (Poisson-gamma-Poisson) and even the Holla (Poisson-inverse Gaussian) distributions, among others. Every text covers the probabilities associated with throwing two dice, but what if one of the dice has a shaved face? Four lines of code are all that is needed to set up and solve this simple, although otherwise extremely tedious, problem.

Chapter 4, “Distributions of Functions of Random Variables” (32 pp.), covers the method of transformations, the moment-generating function approach, products and ratios of random variables, and sums and differences of random variables. All of the mathematical tedium normally associated with such a chapter is completely removed. Therefore, a professor could assign all of the problems from chapter 4 of Hogg and Craig (1995), and the student could concentrate on the nature of transforms without getting bogged down in the mechanics of integration. Note that mathStatica is sufficiently low level that the student has no hope of using it to solve problems unless he understands the material well.

Chapter 5, “Systems of Distributions” (38 pp.), covers the Pearson and Johnson families, Gram–Charlier expansions, Hermite polynomials, nonparametric kernel density estimation, and the method of moments. If one is willing to trust data-based calculations of higher moments, then it is not too much work to redefine both the Pearson and Johnson families based on higher-order polynomials, rather than the usual quadratic ones.

Chapter 6, on multivariate distributions (64 pp.), covers expectations, moments, and generating functions; independence and dependence; the multivariate normal distribution; the multivariate Cauchy; and the multinomial and bivariate Poisson distributions. Calculation of marginal and conditional distributions is quite easy since each requires only a line of code. The same is

true for calculating moments and generating functions. Graphing also is easy using Mathematica's powerful graphical capabilities. A few pages are devoted to copulas; again, these are easy to implement.

Chapter 7 (26 pp.) covers unbiased estimation of population moments, moments of moments, and augmented symmetric and power sums. Moment conversion formulas reappear and are extended to the  $k$  statistics (unbiased estimators of cumulants) and the  $h$  statistics (unbiased estimators of central moments). The power of this book is amply conveyed when the former are easily extended to the generalized  $k$  statistics known as "polykays" (plural of poly- $k$ ); and in an apparent novelty, the latter are extended to "poly-aches" (plural of poly- $h$ ). The accuracy of mathStata/Mathematica is illustrated by revealing errors in long-used formulas in published references (e.g., Eq. 12.70 in Stuart and Ord 1986; the product-cumulant formula  $\kappa(3^2 2^2)$ ). This formula contains some 12 terms, and the correct version (with a demonstration of correctness) is given in this chapter. Examples of published errors in other references occur throughout the book. While testing mathStata, the authors compiled a lengthy list of such errors that is available at the book's website, [www.mathstata.com](http://www.mathstata.com).

Chapter 8, "Asymptotic Distributions" (24 pp.), covers convergence in distribution, asymptotic distribution, the central-limit theorem, convergence in probability, van Beek's bound, and the inequalities of Chebyshev and Markov. A distinct pedagogical advantage of using the computer to teach these concepts is that the student can fiddle with  $n$  and other parameters, and observe the convergence rates. With only a couple of exceptions, all cases considered are i.i.d., so the treatment is at an elementary level.

Chapter 9, "Statistical Decision Theory" (24 pp.), covers loss and risk, mean squared error as risk, and order statistics. With the power of a computer, it becomes easy to compute and graph risk and other such quantities. The mathStata function `OrderStat` automates the calculation of the pdf or joint pdf of an order statistic, and it is used extensively in this chapter.

Chapter 10, on unbiased parameter estimation (24 pp.) covers Fisher information, best unbiased estimators, sufficient statistics, and minimum-variance unbiased estimation, including the Rao-Blackwell theorem. As always, a benefit of mathStata/Mathematica is the elimination of technical drudgery. Example 2 of this chapter, computing the information lost due to censoring of a Poisson variable with parameter  $\lambda$ , is almost trivial. Thus, exploring various aspects of the problem (e.g., graphing relative information against  $\lambda$ ) becomes easy.

Chapter 11, "Principles of Maximum Likelihood Estimation" (30 pp.), covers the likelihood function, maximum likelihood estimation, and properties of maximum likelihood estimators (MLEs). This chapter constitutes preparation for Chapter 12 (42 pp.), which covers use of the `FindMaximum` command and various optimization algorithms, including BFGS and Newton-Raphson. It pays admirable attention to numerical detail. The three basic types of MLE standard errors (Fisher, Hessian, and OPG) are introduced and compared. An extended autoregressive conditional heteroscedasticity (ARCH) example is used illustratively. I can affirm that doing nonlinear estimation is much easier with mathStata than with Mathematica alone.

Various appendices, notes, and references (64 pp.) conclude the text. The entire book, chapter by chapter, is duplicated as Mathematica notebooks. There is no need to read a sequence of commands in the text and then type them into the computer; the reader can open up the relevant chapter notebook and click on the commands. The reader can interactively modify and experiment with the examples in the text. For readers who do not already have it, the book comes with a 30-day trial version of Mathematica. Sixty days would have been better, and unfortunately this trial version also does not allow the user to save worksheets.

Each chapter concludes with several exercises, 101 altogether. A detailed solutions manual is available to instructors of courses using the text. The small number of exercises should not cause concern; any instructor, having seen the exercises and the detailed solutions, would have little trouble conjuring up more exercises. Moreover, any mathematical statistics text will be filled with problems that can be assigned for solution with mathStata.

The preface makes the salient point that "when one is armed with mathStata, the whole notion of what is difficult changes," and this theme recurs throughout the book. Already a couple of examples have been given. Here is one more: Moments of moments are conceptually easy, and for higher orders are notoriously difficult without mathStata. With it, they are no harder than invoking a single command.

This book will be of immediate use to four classes of persons: (1) any person who would do more of the Hogg-Craig-style basics, if only it were much

easier, (2) people who work with real data whose needs are ill met by traditional statistical packages (e.g., actuaries); (3) persons whose computational needs consistently bump into the bounds of traditional software (e.g., econometricians working with esoteric distributions), and (4) teachers who want to innovate the traditional mathematical statistics curriculum.

This book has the potential to revolutionize the teaching of mathematical statistics. Typically, a professor using the text by Hogg and Craig (1995) might choose a few problems from each section and assign them, knowing that tedious mathematical drudgery will occupy much of the student's time. However, if *Mathematical Statistics with Mathematica* is used as a supplement, then the professor can simply ask students to solve all or most of the problems in the chapter. Again, if the student does not know the Hogg and Craig material, then mathStata will be of little use. I expect that in a couple of years, some journal will publish a "report from the field" that describes the collective experience of professors who have chosen to use this book in conjunction with a standard first-year text. Such a report will make for most interesting—and perhaps revolutionary—reading.

While writing an earlier article (McCullough 2000), Internet searches convinced me (no great surprise) that very few persons were using Mathematica for statistical purposes. I shall be equally surprised if the present volume does not change this. These internet searches put me in touch with one of the authors of the book, and I wound up beta-testing mathStata. Other than a review copy of the book, I received nothing other than a wonderful time playing with the included software.

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## Empirical Likelihood.

Art B. OWEN. Boca Raton, FL: Chapman and Hall/CRC, 2001. ISBN 1-58488-071-6. xv + 304 pp. \$74.95.

In recent years, the method of empirical likelihood has attracted tremendous interest in statistics, biostatistics, econometrics, and engineering. Numerous articles have been published since Owen's original articles (Owen 1988, 1990, 1991), which set the basis for all successive developments in this area. *Empirical Likelihood* is the first book on the subject. It not only develops the basic concepts and necessary tools for using empirical likelihood, but also provides many interesting numerical examples.

The empirical likelihood method combines the benefits of likelihood methods with those of estimating equations. It is a nonparametric methodology for dealing with situations where only numerical characteristics of the underlying model are specified. Unlike ordinary likelihood methods, empirical likelihood uses an estimated nonparametric likelihood function in combination with information from a set of estimating equations. This unique feature makes empirical likelihood a useful tool for tackling problems involving a large number of unknown parameters.

The book provides a detailed look at empirical likelihood inference and presents a wide variety of applications. In particular, it describes the advantages and disadvantages of the empirical likelihood method over competing methods. A distinguishing feature of the book is its blend of theory and applications, with an emphasis on the presentation of analyses of real data that nicely demonstrate the usefulness of empirical likelihood methods.

As stated in the Preface, the book is aimed primarily at graduate students, at practitioners looking for new ways to handle data, and at researchers looking for new challenges. A basic background in statistics, including familiarity with likelihood-based methods, and some knowledge of computing is assumed.